

Which method to estimate K_d and K_i in fluorescence polarization?

Methodology and Examples

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Outline

- **Context**
 - Saturation experiment
 - Competition experiment
- **Fluorescence Polarization**
 - Advantages
 - Principle
- **Estimation of K_d in fluorescence polarization**
 - Comparison of models
 - Examples & Simulations
- **Estimation of K_i in fluorescence polarization**
 - Comparison of models
 - Examples & Simulations
- **Conclusion**



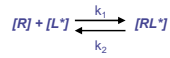
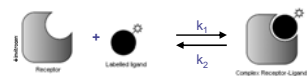
Context: Ligand Binding Assay

- A binding assay is used to measure the amount of binding or affinity between two molecules. There are numerous types of ligand binding assays.
- Widely applied in life science
- **Examples**
 - Targeting protein-protein interactions
 - Protein-protein interaction (PPI) can contribute to many diseases, including cancer, and plays a key role in maintaining the malignant phenotype in tumor cells. Selective, small-molecule modulation of PPIs is therefore an area of interest to pharmaceutical science.
- Two basic steps: saturation and competition experiments



Two basic types of ligand binding experiment 1 – Saturation experiment

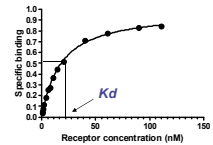
In saturation binding assays, a labelled ligand, $[L^*]$, binds to a receptor, $[R]$



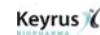
- Conservation of mass requires :
 $[L^*]_0 = [L^*] + [RL^*]$ and $[R]_0 = [R] + [RL^*]$
 with $[L^*]_0$, total concentration of labelled ligand
 $[R]_0$, total concentration of receptor

- At equilibrium: $k_1 [L^*] [R] = k_2 [RL^*]$

$$K_d = \frac{k_2}{k_1} = \frac{[L^*] [R]}{[L^*R]}$$

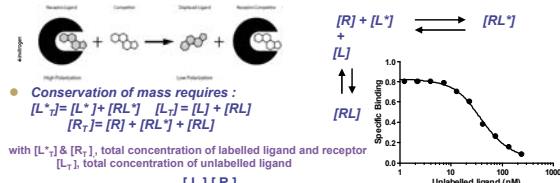


- The equilibrium dissociation constant (K_d) provides a measure of affinity between the receptor $[R]$ and the labelled ligand $[L^*]$



Two basic types of ligand binding experiment 2 – Competition experiment

In competitive binding assays, an unlabelled ligand (competitor), [L], binds to a receptor [R] in competition with a labelled ligand, [L*].



- Conservation of mass requires :
 $[L^*]_T = [L^*] + [RL^*]$ $[L]_T = [L] + [RL]$
 $[R]_T = [R] + [RL^*] + [RL]$

with $[L^*]_T$ & $[R]_T$, total concentration of labelled ligand and receptor
 $[L]_T$, total concentration of unlabelled ligand

- At equilibrium: $K_i = \frac{[L][R]}{[LR]}$

- When the K_d of the labelled ligand $[L^*]$ is known, the equilibrium inhibition constant (K_i) provides an indirect measure of affinity between receptor $[R]$ and unlabelled ligand $[L]$



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Why use Fluorescence Polarization?

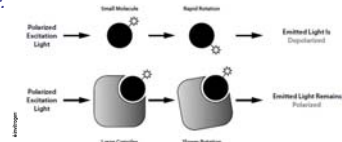
- Previously, traditional radioligand binding experiments were widely used for saturation and competition experiments
- Fluorescence Polarization (FP) is one alternative to the traditional binding radioligand experiments, in particular for small molecules
 - Application areas of FP:
 - Protein/Protein interactions
 - Enzyme/Substrate
 - Antigen/Antibodies
 - ...
- Fluorescence Polarization (FP) offers numerous advantages over the more conventional methods:
 - Can be used in HTS
 - Faster and highly reproducible results (low variabilities intra/inter experiment)
 - No separation of bound and free ligand required
 - No radioactive materials



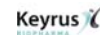
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FP principle and signal expression

- Fluorescence Polarization (FP) assays are based on measuring the polarization (P) of light caused by changes in molecular size.
- The rotational speed of a molecule is decreased once it is bound to a receptor.



- Polarized emission can be measured by polarization and anisotropy, r .



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Relationship between anisotropy (r) and F_{bound}

- In FP, there is no necessity for separation of bound and free ligand
- The fraction of bound to free, F_{bound} , is directly deduced from the anisotropy, r, using the equation:

$$F_{bound} = \frac{\text{binding_site_occupied}}{\text{total_ligand}} = \frac{[RL^*]}{[L^*]} = \frac{r - r_{free}}{(r_{bound} - r)Q + (r - r_{free})}$$

Where r_{free} is the anisotropy of the free labelled ligand,
 r_{bound} is the anisotropy of the ligand-protein complex at saturation and
 Q is the ratio of fluorescence intensities of bound versus free ligand

- The binding of a ligand to a receptor is expressed by the fraction of bound receptors F_{bound}



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Outline

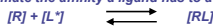
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- Estimation of Ki in fluorescence polarization**
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Estimation of Kd in saturation experiment in FP

- $F_{bound} = (\text{binding sites occupied}) / (\text{total ligand}) = [RL^*] / [L^*]$
- F_{bound} allows to estimate the affinity a ligand has to a receptor (Kd):



$$Kd = [R][L^*] / [RL^*]$$

Case 1: Specific case

$[R_T] \gg [L^*] \rightarrow [R] \approx [R_T]$
 • Assumption of no receptor depletion

$$F_{bound} = \frac{[RL^*]}{[L^*]} = \frac{[R_T]}{Kd + [R_T]}$$

Case 2: General case

$[R_T] \text{ not } \gg [L^*] \rightarrow [R] = [R_T] - [RL^*]$

$$F_{bound} = \frac{[RL^*]}{[L^*]} = \frac{a - \sqrt{a^2 - 4[R_T][L^*]}}{2[L^*]}$$

with $a = (Kd + [R_T] + [L^*])$



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Examples of estimations of Kd in FP

- In FP experimental assay, when can we assume the assumption of no receptor depletion?

- Example 1:**
 - $[L^*] = 1.5 \text{ nM}$
 - $[R_T]$ is between 0.002 nM to 257.2 nM
- Example 2:**
 - $[L^*] = 10 \text{ nM}$
 - $[R_T]$ is between 2.5 to 1280 nM

Examples	Kd [95%CI] (nM)	Kd [95%CI] (nM)
	Case 1: $[R] = [R_T]$ No receptor depletion	Case 2: $[R] = [R_T] - [RL^*]$
Example 1	1.417 [1.230 ; 1.604]	0.702 [0.607 ; 0.798]
Example 2	7.336 [5.774, 8.898]	3.230 [1.880, 4.581]

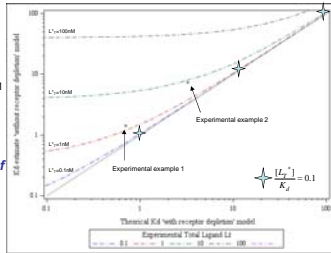
- In those examples, there is a factor around 2 between both methods to estimate the equilibrium dissociation constant Kd.



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Simulations for Kd estimation

- Design**
 - The FP were generated using receptor depletion hypothesis model (case 2) with the following factors:
 - [R₀] concentrations: 2.10⁴ to 1.10⁵ nM
 - [L_γ] concentrations: [0.1 1 10 100] nM
 - Theoretical K_d from 0.1nM to 100nM
- Conclusion: providing [L_γ^{*}] < 10% of K_d, we can assume that [R] ≈ [R_γ]**

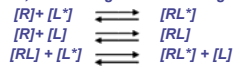


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Estimation of K_i in competition experiment

- The affinity of the competitor, unlabelled ligand [L], for the receptor [R] can be indirectly determined by measuring its ability to compete with, and thus inhibit, the binding of a labelled ligand [L*] to its receptor**



- The affinity of the unlabelled ligand for the receptor can be obtained from different methods:**
 - Estimation of constant of inhibition (K_i)
 - The Cheng-Prusoff equation
 - The Nikolovska-Coleska & all equation
 - ...
 - Direct estimation of the affinity of the competitor (K_{d2})
 - The Wang exact mathematical expression

Methods based on equation

- Shared assumptions**
 - A single class of receptor binding sites
 - Assay is at equilibrium
- Cheng-Prusoff equation**

$$K_i = \frac{IC_{50}}{\left(\frac{[L^*]}{K_d} + 1\right)}$$

Where [L*] = concentration of labelled ligand
K_d = dissociation constant of labelled ligand
- Nikolovska-Coleska & al. equation**
 - [R] = [R₀] - [RL]
 - [L] = [L₀] - [RL]
$$K_i = \frac{[L_{50}]}{\left(\frac{[L_{50}]}{K_d} + [R_0] + 1\right)}$$

Where [R₀] = Initial concentration of receptor
[L] = Concentration of unlabelled ligand
[R₀] = C - (K_d + [L₀] - [R₀]) + (K_d + [L₀] - [R₀])² + 4[R₀](K_d)² / 2
[R₀] = ([R₀] - [R₀])
[R₀] = ([R₀] - [R₀]) / 2
[L₅₀] = [L₀] - [R₀]
[L₅₀] = K_d - [R₀] + [R₀] + K_d / ([L₀] - [R₀])

Wang model: An exact analytical treatment of competitive binding

- Wang has described an exact expression of competitive binding:



- After substitution and rearrangement, this leads to the following equation:

$$F_{bound} = \frac{[R_T]}{[L_T]} = \frac{2 \cdot \sqrt{(a^2 - 3b)} \cdot \cos\left(\frac{\theta}{3}\right) - a}{3K_{d1} + 2 \cdot \sqrt{(a^2 - 3b)} \cdot \cos\left(\frac{\theta}{3}\right) - a}$$

Where $a = K_{d1} + K_{d2} + [L_T] + [L_T^*] - [R_T]$
 $b = K_{d1} \cdot ([L_T^*] - [R_T]) + K_{d2} \cdot ([L_T] - [R_T]) + K_{d1} \cdot K_{d2}$
 $c = -K_{d1} \cdot K_{d2} \cdot [R_T]$

- $[R_T]$ and $[L_T^*]$ are the experimental constants
- K_{d1} must have been previously obtained from a direct binding experiment

Comparison of Ki

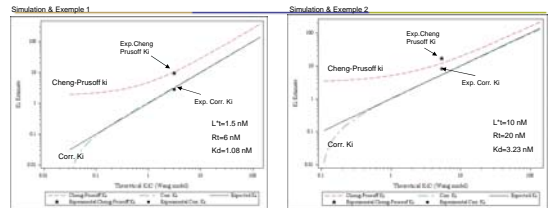
- Example 1:**
 - $[L_T^*] = 1.5$ nM and $[R_T] = 6$ nM
 - $[L_T]$ is between 0.51 nM to 10000 nM
 - $K_{d1} = 1.08$ nM and $IC_{50} = 22.3$ [17.5 ; 28.6] nM
- Example 2:**
 - $[L_T^*] = 10.0$ nM and $[R_T] = 20.0$ nM
 - $[L_T]$ is between 0.57 to 238 nM
 - $K_{d1} = 3.23$ nM and $IC_{50} = 68.1$ [40.1 ; 115.3] nM

Experiment	Ki Cheng Prusoff	Corrected Ki Nikolovska-Coleska & al.	Kd2 Wang model
Example 1	9.35	2.81	3.14 [2.395 ; 3.888]
Example 2	16.62	7.33	5.34 [3.896 ; 6.778]

Simulations of competition experiment (1)

- Design**
 - FP were generated using the Wang model under 2 experimental conditions with the following factors:
 - $[L_T^*]$ is between 0.1 nM to 10000 nM
 - Theoretical K_{d2} of competitor from 0.1 nM to 10 nM
 - 1st simulation (Example 1):**
 - $[L_T^*] = 1.5$ nM $[R_T] = 6$ nM and $K_{d1} = 1.08$ nM
 - 2nd simulation (Example 2):**
 - $[L_T^*] = 10$ nM $[R_T] = 20$ nM and $K_{d1} = 3.23$ nM
- EC_{50} , Cheng-Prusoff K_i and $Coor. K_i$ (Nikolovska-Coleska & al.) were estimated.

Simulations of competition experiment (2)



- In these experimental conditions:**
 - the Cheng-Prusoff K_i was over estimated
 - The $Coor. K_i$ was:
 - always lower than the Cheng-Prusoff K_i
 - very close to the affinity of the competitor simulated with the Wang model (K_{d2})

Conclusion

- In FP, classical binding models need to be adapted to the Fbound
- Which method to estimate Kd in FP?
 - The classical model, $[R] \approx [R_T]$, is not well adapted: over estimation of Kd
 - The general model, taken into account the receptor depletion $[R] = [R_T] - [RL]$, must be used in FP experiments
- Which method to estimate Ki in FP?
 - Ki calculated with Cheng-Prusoff biased the affinity of a competitor for a receptor
 - The Nikolovska-Coleska equation (corr. Ki) and the exact analytical treatment of competitive binding (Wang model) are equivalent methods:
 - Advantage of Wang model : 95% confidence interval is estimated
 - Advantage of Corr. Ki : equation for uncompetitive inhibition is available
 - Both methods: difficult to generalize in particular cases as dimeric models

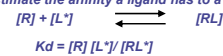
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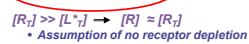
Back-up slides

Estimation of Kd in saturation experiment in FP

- $F_{bound} = (\text{binding sites occupied}) / (\text{total ligand}) = [RL^*] / [L^*]_T$
- F_{bound} allows to estimate the affinity a ligand has to a receptor (Kd):

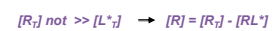


Case 1: Specific case



$$F_{bound} = \frac{[RL^*]}{[L^*]_T} = \frac{[R_T][L^*]}{K_d + [R_T]}$$

Case 2: General case



$$F_{bound} = \frac{[RL^*]}{[L^*]_T} = \frac{a - \sqrt{a^2 - 4[R_T][L^*]_T}}{2[L^*]_T}$$

$$\text{with } a = (K_d + [R_T] + [L^*]_T)$$



Estimation of Kd in saturation experiment in FP

Case 1: Specific case

- Assumption of no receptor depletion

$$K_d = \frac{[L^*][R]}{[RL]} \quad [R] = [R_T] \\ [L^*] = [L_T] - [RL]$$

$$K_d = \frac{([L_T] - [RL])[R_T]}{[RL]}$$

$$K_d + [R_T] = \frac{([L_T] - [RL])[R_T]}{[RL]} + [R_T] \implies K_d + [R_T] = \frac{([L_T] - [RL])[R_T] + [RL][R_T]}{[RL]}$$

$$K_d + [R_T] = \frac{[L_T][R_T] - [RL][R_T] + [RL][R_T]}{[RL]}$$

$$K_d + [R_T] = \frac{[L_T][R_T]}{[RL]}$$

$$F_{bound} = \frac{[RL^*]}{[L_T^*]} = \frac{[R_T]}{K_d + [R_T]}$$

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Estimation of Kd in saturation experiment in FP

Case 2: General case

- Assumption of receptor depletion

$$K_d = \frac{[L^*][R]}{[RL]} \quad [R] = [R_T] - [RL^*] \\ [L^*] = [L_T] - [RL^*]$$

$$K_d = \frac{([L_T] - [RL^*])[R_T - [RL^*]]}{[RL^*]} \implies K_d[RL^*] = ([L_T] - [RL^*])([R_T] - [RL^*])$$

$$K_d[RL^*] - ([L_T] - [RL^*])([R_T] - [RL^*]) = 0 \implies -[RL^*]^2 + (K_d + [R_T] + [L_T])[RL^*] - [R_T][L_T] = 0$$

$$[RL^*] = \frac{(K_d + [R_T] + [L_T]) - \sqrt{(K_d + [R_T] + [L_T])^2 - 4[R_T][L_T]}}{2}$$

$$[RL^*] = \frac{(K_d + [R_T] + [L_T]) - \sqrt{(K_d + [R_T] + [L_T])^2 - 4[R_T][L_T]}}{2}$$

$$F_{bound} = \frac{[RL^*]}{[L_T^*]} = \frac{(K_d + [R_T] + [L_T]) - \sqrt{(K_d + [R_T] + [L_T])^2 - 4[R_T][L_T]}}{2[L_T^*]}$$

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Wang model: An exact analytical treatment of competitive binding (1)

- Wang has described an exact expression of competitive binding:

$$R + L^* \xrightleftharpoons{K_{d1}} RL^* \quad R + L \xrightleftharpoons{K_{d2}} RL \quad [R_T] = [RL^*] + [RL] + [R] \\ K_{d1} = \frac{[R][L^*]}{[RL^*]} \quad K_{d2} = \frac{[R][L]}{[RL]} \quad \text{Conservation of mass requires: } [L_T^*] = [RL^*] + [L^*] \\ [L_T] = [RL] + [L] \quad (\text{Eq. 3})$$

- Expressing $[RL^*]$ and $[RL]$ as function of total ligand (L^*) and total competitor concentrations (L_T) yield Eq.4 and 5

$$\implies RL^* = \frac{[R][L_T^*]}{K_{d1} + [R]} \quad (\text{Eq. 4}) \quad RL = \frac{[R][L_T]}{K_{d2} + [R]} \quad (\text{Eq. 5})$$

- Substitution of Eq.4 and 5 in Eq. 3, yields Eq. 6, which after rearrangement corresponds to the cubic Eq. 7

$$[R_T] = \frac{[R][L_T^*]}{K_{d1} + [R]} + \frac{[R][L_T]}{K_{d2} + [R]} + [R] \quad (\text{Eq. 6})$$

$$[R]^3 + a[R]^2 + b[R] + c = 0 \quad (\text{Eq. 7}) \quad \text{Where } a = K_{d1} + K_{d2} + [L_T] + [L_T^*] - [R_T] \\ b = K_{d2}([L_T] - [R_T]) + K_{d1}([L_T] - [R_T]) + K_{d1} \cdot K_{d2} \\ c = -K_{d1} \cdot K_{d2} \cdot [R_T]$$

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Wang model: An exact analytical treatment of competitive binding (2)

- The only meaningful solution of Eq.7 can be written as in Eq.8 and the expression of is given in Eq. 9.

$$[R] = \frac{a}{3} + \frac{2}{3} \sqrt{(a^2 - 3b)} \cos \frac{\theta}{3} \quad \text{where } \theta = \arccos \left[\frac{-2a^3 + 9ab - 27c}{2 \sqrt{(a^2 - 3b)^3}} \right] \quad (\text{Eq. 8})$$

$$\frac{[RL^*]}{[L_T^*]} = \frac{[R]}{K_{d1} + [R]} = \frac{2 \sqrt{(a^2 - 3b)} \cos \frac{\theta}{3} - a}{3K_{d1} + 2 \sqrt{(a^2 - 3b)} \cos \frac{\theta}{3} - a} \quad (\text{Eq. 9})$$

Where

$$a = K_{d1} + K_{d2} + [L_T] + [L_T^*] - [R_T] \\ b = K_{d2}([L_T] - [R_T]) + K_{d1}([L_T] - [R_T]) + K_{d1} \cdot K_{d2} \\ c = -K_{d1} \cdot K_{d2} \cdot [R_T]$$

- $[R_T]$ and $[L_T]$ are the experimental constants
- $Kd1$ must have been previously obtained from a direct binding experiment

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