

Quantitative assessment of drug interactions by linear mixed effects modeling

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Interaction index

- Quantitative method for assessment of interaction effects of two drugs

- Definition based on Loewe Additivity Model:

$$\tau = \frac{d_1}{D_{y,1}} + \frac{d_2}{D_{y,2}} \begin{cases} < 1, \text{ synergy} \\ = 1, \text{ additivity} \\ > 1, \text{ antagonism} \end{cases}$$

- d_1, d_2 : doses of drugs 1 and 2 that in combination produce effect y (known)
- $D_{y,1}, D_{y,2}$: doses of drugs 1 and 2 that produce same effect if applied singly (unknown)
- $D_{y,1}$: inverse of dose-response curve for drug 1
- $D_{y,2}$: inverse of dose-response curve for drug 2

Research question

Is interaction effect of two drugs additive, synergistic or antagonistic ?

Statistical inference

- Model dose-response curve for drug i ($i = 1, 2$) by fitting two-parameter log-logistic model to normalized effect $y \in (0, 1)$:

$$y = \frac{1}{1 + \left(\frac{D_{y,i}}{D_{m,i}} \right)^{b_i}}$$

$D_{y,i}$: dose of drug i , producing effect y

$D_{m,i}$: median effective dose of drug i (e.g. EC_{50} , LD_{50})

b_i : Hill slope (for drug i)

- Transfer model into median-effect equation (Chou and Talalay, *Advances in Enzyme Regulation*, 1984):

$$\frac{y}{1-y} = \left(\frac{D_{y,i}}{D_{m,i}} \right)^{-b_i}$$

- Two analysis approaches (Lee and Kong, *Stat Biopharm Res*, 2009)

Global assessment approach (1)

Assumption: two drugs combined in ,fixed dose ratio':

$$\frac{d_1}{d_2} = \frac{\omega_1}{\omega_2}, d_1 + d_2 = D_C, D_C = (d_1, d_2)$$

1. Taking log() of median-effect equation yields **simple linear regression model**:

$$\log\left(\frac{y}{1-y}\right) = -b_i \cdot (\log(D_{y,i}) - \log(D_{m,i})) =: \beta_{0,i} + \beta_{1,i} \cdot \log(D_{y,i}), \text{ with } \text{median-effect equation:}$$

$$\beta_{0,i} = b_i \cdot \log(D_{m,i}), \beta_{1,i} = -b_i$$

$$\frac{y}{1-y} = \left(\frac{D_{y,i}}{D_{m,i}}\right)^{-b_i}$$

2. For drugs $i = 1, 2$: Regress $\log(y/(1-y))$ on $\log(D_{y,i})$ to estimate $\beta_{0,i}$ and $\beta_{1,i}$.

3. Estimation of τ :

$$\hat{\tau} = \frac{d_1}{\hat{D}_{y,1}} + \frac{d_2}{\hat{D}_{y,2}} \text{ with } \hat{D}_{y,i} = \exp\left(-\frac{\hat{\beta}_{0,i}}{\hat{\beta}_{1,i}}\right) \cdot \left(\frac{y}{1-y}\right)^{1/\hat{\beta}_{1,i}} \quad (i = 1, 2)$$

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Global assessment approach (3)

7. Define grid of effects y .

8. For each effect y :

$$\hat{\tau}_{GA} = \frac{\hat{D}_{y,C} \frac{\omega_1}{\omega_1 + \omega_2}}{\hat{D}_{y,1}} + \frac{\hat{D}_{y,C} \frac{\omega_2}{\omega_1 + \omega_2}}{\hat{D}_{y,2}}$$

(a) Use **modified estimator** $\hat{\tau}_{GA}$ to estimate interaction index τ .

(b) Apply **delta method** to compute approximative variance for $\hat{\tau}_{GA}$:

$$\text{Var}(\hat{\tau}_{GA}) = \left(\frac{\omega_1}{\omega_1 + \omega_2} \frac{\hat{D}_{y,C}}{\hat{D}_{y,1}}\right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,1})}{\hat{\beta}_{1,1}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,1}, \hat{\beta}_{1,1}) \cdot \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,1}\right)}{\hat{\beta}_{1,1}^3} + \frac{\text{Var}(\hat{\beta}_{1,1}) \cdot \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,1}\right)^2}{\hat{\beta}_{1,1}^4} \right]$$

$$+ \left(\frac{\omega_2}{\omega_1 + \omega_2} \frac{\hat{D}_{y,C}}{\hat{D}_{y,2}}\right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,2})}{\hat{\beta}_{1,2}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,2}, \hat{\beta}_{1,2}) \cdot \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,2}\right)}{\hat{\beta}_{1,2}^3} + \frac{\text{Var}(\hat{\beta}_{1,2}) \cdot \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,2}\right)^2}{\hat{\beta}_{1,2}^4} \right]$$

$$+ \left(\frac{\hat{D}_{y,C}}{\omega_1 + \omega_2} \cdot \left(\frac{\omega_1}{\hat{D}_{y,1}} + \frac{\omega_2}{\hat{D}_{y,2}}\right)\right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,C})}{\hat{\beta}_{1,C}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,C}, \hat{\beta}_{1,C}) \cdot \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,C}\right)}{\hat{\beta}_{1,C}^3} + \frac{\text{Var}(\hat{\beta}_{1,C}) \cdot \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,C}\right)^2}{\hat{\beta}_{1,C}^4} \right]$$

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Global assessment approach (2)

4. Make use of ,fixed dose ratio' assumption to modify equation for τ :

$$d_1 = D_C \frac{\omega_1}{\omega_1 + \omega_2}, d_2 = D_C \frac{\omega_2}{\omega_1 + \omega_2}$$

$$\frac{d_1}{d_2} = \frac{\omega_1}{\omega_2}, d_1 + d_2 = D_C, D_C = (d_1, d_2)$$

'fixed dose ratio' assumption:

5. Estimate combination dose D_C at effect y :

$$\hat{D}_{y,C} = \exp\left(-\frac{\hat{\beta}_{0,C}}{\hat{\beta}_{1,C}}\right) \cdot \left(\frac{y}{1-y}\right)^{1/\hat{\beta}_{1,C}}$$

$$\hat{\tau} = \frac{d_1}{\hat{D}_{y,1}} + \frac{d_2}{\hat{D}_{y,2}}$$

6. **Modified estimator** for τ :

$$\hat{\tau}_{GA} = \frac{\hat{D}_{y,C} \frac{\omega_1}{\omega_1 + \omega_2}}{\hat{D}_{y,1}} + \frac{\hat{D}_{y,C} \frac{\omega_2}{\omega_1 + \omega_2}}{\hat{D}_{y,2}}$$

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Global assessment approach (4)

(c) Calculate approximative $(1-\alpha)$ confidence interval for τ :

$$\hat{\tau}_{GA} \cdot \exp\left(\pm \frac{t_{n+n_c-6, \alpha/2}}{\hat{\tau}_{GA}} \cdot \sqrt{\text{Var}(\hat{\tau})}\right)$$

$$n = \sum_{i=1}^2 n_i; n_i: \# \text{ of observations when drug } i \text{ is applied alone}$$

n_c : # of observations when combination of two drugs is applied

9. Plot: effects y vs. estimated interaction indices $\hat{\tau}_{GA}$

10. Computation of pointwise $(1-\alpha)$ confidence bound for curve of effects vs. interaction indices.

R code provided by Lee and Kong at:

<http://biostatistics.mdanderson.org/SoftwareDownload/>.

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Drawback

Global assessment approach:

Assumes that all data were collected from a single dose-response experiment.

9

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Practical situations:

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Naive solution

- (1) **Merge** data of all dose-response experiments.
- (2) Apply global assessment approach to **merged** data set.

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Problem

Recall: Approximative variance of estimated interaction index:

$$\begin{aligned} \text{Var}(\hat{\tau}_{G,i}) &= \left(\frac{\omega_1}{\omega_1 + \omega_2} \cdot \frac{\hat{D}_{y,1}}{\hat{D}_{y,1}} \right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,1})}{\hat{\beta}_{1,1}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,1}, \hat{\beta}_{1,1}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,1} \right)}{\hat{\beta}_{1,1}^3} + \frac{\text{Var}(\hat{\beta}_{1,1}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,1} \right)^2}{\hat{\beta}_{1,1}^4} \right] \text{Var}(\hat{D}_{y,1}) \\ &+ \left(\frac{\omega_2}{\omega_1 + \omega_2} \cdot \frac{\hat{D}_{y,2}}{\hat{D}_{y,2}} \right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,2})}{\hat{\beta}_{1,2}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,2}, \hat{\beta}_{1,2}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,2} \right)}{\hat{\beta}_{1,2}^3} + \frac{\text{Var}(\hat{\beta}_{1,2}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,2} \right)^2}{\hat{\beta}_{1,2}^4} \right] \text{Var}(\hat{D}_{y,2}) \\ &+ \left(\frac{\hat{D}_{y,C}}{\omega_1 + \omega_2} \cdot \left(\frac{\omega_1}{\hat{D}_{y,1}} + \frac{\omega_2}{\hat{D}_{y,2}} \right) \right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,C})}{\hat{\beta}_{1,C}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,C}, \hat{\beta}_{1,C}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,C} \right)}{\hat{\beta}_{1,C}^3} + \frac{\text{Var}(\hat{\beta}_{1,C}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,C} \right)^2}{\hat{\beta}_{1,C}^4} \right] \text{Var}(\hat{D}_{y,C}) \end{aligned}$$

- Accounts for within-experiment variability (by variance terms $\text{Var}(\hat{D}_{y,i})$, $i = 1, 2, C$)
- Does not account for between-experiment variability !!!
- Large between-experiment variability can have great impact on estimation of interaction index

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Proposal (2)

(3) Plug in mixed effects model estimates of $\beta_{0,i}$, $\beta_{1,i}$, $\text{Var}(\beta_{0,i})$, $\text{Var}(\beta_{1,i})$ and $\text{Cov}(\beta_{0,i}, \beta_{1,i})$ ($i = 1, 2, C$) into formulas for $\hat{\tau}_{G,i}$ and $\text{Var}(\hat{\tau}_{G,i})$, to yield reliable estimates of τ :

$$\begin{aligned} \hat{\tau}_{G,i} &= \frac{\hat{D}_{y,C} \frac{\omega_1}{\omega_1 + \omega_2} + \frac{\hat{D}_{y,C}}{\hat{D}_{y,1}} \frac{\omega_2}{\omega_1 + \omega_2}}{\frac{\hat{D}_{y,1}}{\hat{D}_{y,1}} + \frac{\hat{D}_{y,2}}{\hat{D}_{y,2}}} \cdot \hat{D}_{y,j} = \exp\left(-\frac{\hat{\beta}_{0,i}}{\hat{\beta}_{1,i}}\right) \cdot \left(\frac{y}{1-y}\right)^{1/\hat{\beta}_{1,i}} \\ \text{Var}(\hat{\tau}_{G,i}) &= \left(\frac{\omega_1}{\omega_1 + \omega_2} \cdot \frac{\hat{D}_{y,C}}{\hat{D}_{y,1}} \right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,1})}{\hat{\beta}_{1,1}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,1}, \hat{\beta}_{1,1}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,1} \right)}{\hat{\beta}_{1,1}^3} + \frac{\text{Var}(\hat{\beta}_{1,1}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,1} \right)^2}{\hat{\beta}_{1,1}^4} \right] \\ &+ \left(\frac{\omega_2}{\omega_1 + \omega_2} \cdot \frac{\hat{D}_{y,C}}{\hat{D}_{y,2}} \right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,2})}{\hat{\beta}_{1,2}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,2}, \hat{\beta}_{1,2}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,2} \right)}{\hat{\beta}_{1,2}^3} + \frac{\text{Var}(\hat{\beta}_{1,2}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,2} \right)^2}{\hat{\beta}_{1,2}^4} \right] \\ &+ \left(\frac{\hat{D}_{y,C}}{\omega_1 + \omega_2} \cdot \left(\frac{\omega_1}{\hat{D}_{y,1}} + \frac{\omega_2}{\hat{D}_{y,2}} \right) \right)^2 \left[\frac{\text{Var}(\hat{\beta}_{0,C})}{\hat{\beta}_{1,C}^2} + \frac{2 \cdot \text{Cov}(\hat{\beta}_{0,C}, \hat{\beta}_{1,C}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,C} \right)}{\hat{\beta}_{1,C}^3} + \frac{\text{Var}(\hat{\beta}_{1,C}) \left(\log\left(\frac{y}{1-y}\right) - \hat{\beta}_{0,C} \right)^2}{\hat{\beta}_{1,C}^4} \right] \end{aligned}$$

(4) Modify R code provided by Lee and Kong accordingly.

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Proposal (1)

(1) **Merge** data of all dose-response experiments.

(2) **Global assessment approach:** For each drug (combination), replace (fixed effects) simple linear regression model

$$z_j = \beta_0 + \beta_1 \cdot d_j + \varepsilon_j, \quad \varepsilon_j \sim N(0, \sigma^2)$$

$$d_j : \log(\text{dose}) \text{ for observation } j = 1, \dots, N = \sum_{k=1}^K n_k, \quad n_k : \# \text{ of observations in experiment } k = 1, \dots, K$$

$$z_j = \log\left(\frac{y_j}{1-y_j}\right), \quad y_j \in (0,1)$$

by **linear mixed effects model**

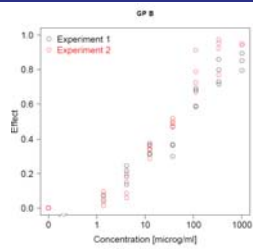
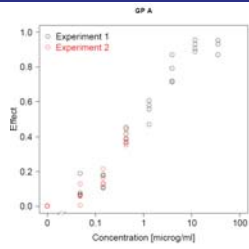
$$z_{kl} = \beta_0 + \beta_1 \cdot d_{kl} + b_{0,k} + b_{1,k} \cdot d_{kl} + \varepsilon_{kl}, \quad \varepsilon_{kl} \sim N(0, \sigma^2), \quad \begin{pmatrix} b_{0,k} \\ b_{1,k} \end{pmatrix} \sim N(0, \Sigma), \quad k = 1, \dots, K, \quad l = 1, \dots, n_k$$

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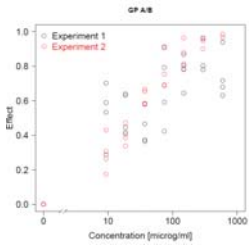
Application to cancer research study

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Data



GP: glycoprotein



- Large within-experiment variability of replicate measurements per concentration (especially for experiment 1)

- Large between-experiment variability (especially for GP B and the combination GP A/B)

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Parameter estimates

Situation	$\hat{\beta}_0$		$\hat{\beta}_1$	
	Fixed Effects Model	Mixed Effects Model*	Fixed Effects Model	Mixed Effects Model*
GP A	0.0812	0.0804	0.8876	0.8864
GP B	-2.9581	-2.9330	0.8042	0.8050
GP A/B	-2.1628	-2.3112	0.7290	0.7732

- **Small difference in parameter estimates** between fixed effects and mixed effects modeling approach

* random intercept + random slope

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Parameter estimates

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* random intercept + random slope

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Parameter estimates/variances

Situation	$\hat{\beta}_0$ (Var)		$\hat{\beta}_1$ (Var)	
	Fixed Effects Model	Mixed Effects Model*	Fixed Effects Model	Mixed Effects Model*
GP A	0.0812 (0.0055)	0.0804 (0.0053)	0.8876 (0.0012)	0.8864 (0.0044)
GP B	-2.9581 (0.0324)	-2.9330 (0.1875)	0.8042 (0.0019)	0.8050 (0.0285)
GP A/B	-2.1628 (0.1528)	-2.3112 (1.5555)	0.7290 (0.0079)	0.7732 (0.1200)

- **Small difference in parameter estimates** between fixed effects and mixed effects modeling approach

* random intercept + random slope

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Parameter estimates/variances/covariances

Situation	$\hat{\beta}_0$ (Var)		$\hat{\beta}_1$ (Var)		$Cov(\hat{\beta}_0, \hat{\beta}_1)$	
	Fixed Effects Model	Mixed Effects Model*	Fixed Effects Model	Mixed Effects Model*	Fixed Effects Model	Mixed Effects Model*
GPA	0.0812 (0.0055)	0.0804 (0.0053)	0.8876 (0.0012)	0.8864 (0.0044)	-0.0002	-0.0003
GP B	-2.9581 (0.0324)	-2.9330 (0.1875)	0.8042 (0.0019)	0.8050 (0.0285)	-0.0068	-0.0717
GP A/B	-2.1628 (0.1528)	-2.3112 (1.5555)	0.7290 (0.0079)	0.7732 (0.1200)	-0.0331	-0.4303

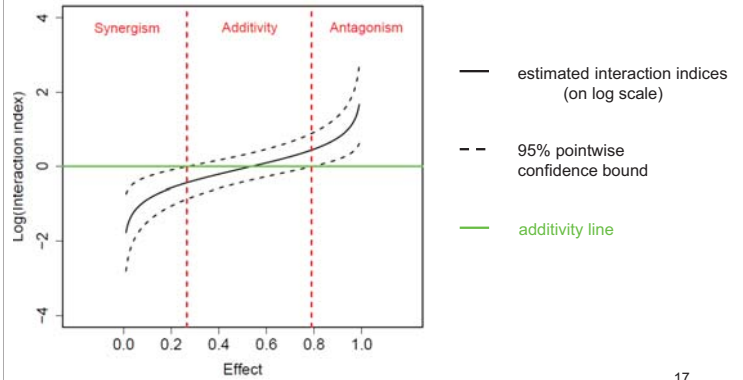
- Small difference in parameter estimates between fixed effects and mixed effects modeling approach

* random intercept + random slope

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Plot of effects vs. log(interaction indices): artificial data

Grid of effects: $y \in [0.01; 0.99]$, stepsize: $\Delta = 0.005$



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Parameter estimates/variances/covariances

Situation	$\hat{\beta}_0$ (Var)		$\hat{\beta}_1$ (Var)		$Cov(\hat{\beta}_0, \hat{\beta}_1)$	
	Fixed Effects Model	Mixed Effects Model*	Fixed Effects Model	Mixed Effects Model*	Fixed Effects Model	Mixed Effects Model*
GPA	0.0812 (0.0055)	0.0804 (0.0053)	0.8876 (0.0012)	0.8864 (0.0044)	-0.0002	-0.0003
GP B	-2.9581 (0.0324)	-2.9330 (0.1875)	0.8042 (0.0019)	0.8050 (0.0285)	-0.0068	-0.0717
GP A/B	-2.1628 (0.1528)	-2.3112 (1.5555)	0.7290 (0.0079)	0.7732 (0.1200)	-0.0331	-0.4303

- Small difference in parameter estimates between fixed effects and mixed effects modeling approach

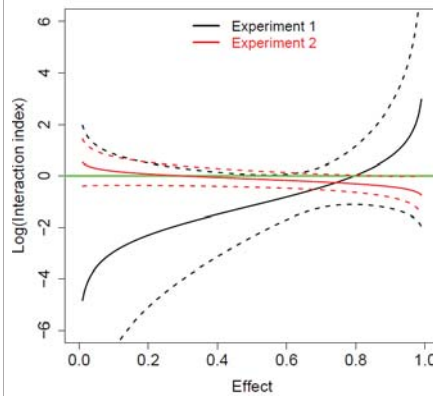
- Fixed effects modeling approach underestimates variances/covariances of parameter estimates

* random intercept + random slope

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Plot of effects vs. log(interaction indices): study data

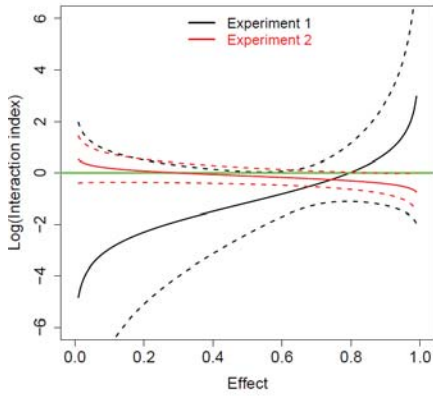
Grid of effects: $y \in [0.01; 0.99]$, stepsize: $\Delta = 0.005$



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Plot of effects vs. log(interaction indices): study data

Grid of effects: $y \in [0.01;0.99]$, stepsize : $\Delta = 0.005$

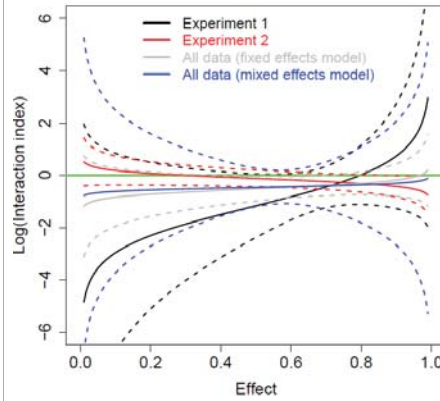


- Large difference between experiments

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Plot of effects vs. log(interaction indices): study data

Grid of effects: $y \in [0.01;0.99]$, stepsize : $\Delta = 0.005$

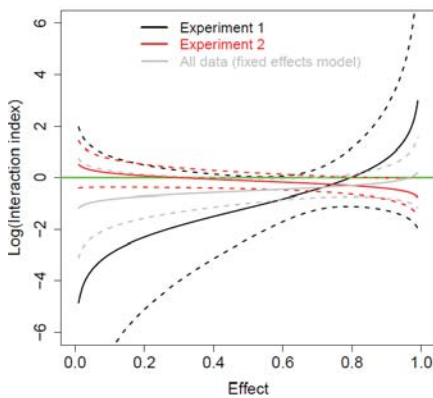


- Large difference between experiments
- Fixed/mixed effects modeling approach result in similar estimates of interaction index
- 95% confidence bound wider for mixed effects modeling approach

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Plot of effects vs. log(interaction indices): study data

Grid of effects: $y \in [0.01;0.99]$, stepsize : $\Delta = 0.005$

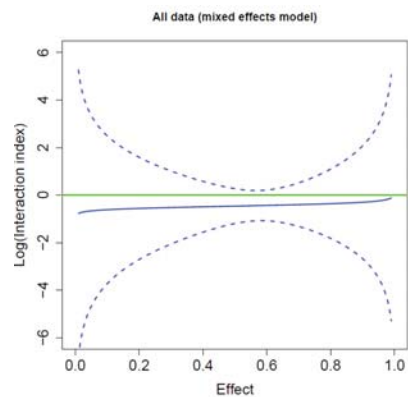


- Large difference between experiments

18

Plot of effects vs. log(interaction indices): study data

Grid of effects: $y \in [0.01;0.99]$, stepsize : $\Delta = 0.005$



- Concentration-independent interaction effect of two glycoproteins
- Slight synergy tendency at all concentrations
- No statistically significant deviation from additivity assumption ($\alpha=5\%$)

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Discussion

- Global assessment approach (Lee and Kong, 2009) allows quantitative assessment of drug interactions for complete dose range.
- Drawback: approach assumes that all data were collected from a **single** dose-response experiment
- If more than one experiment:
 - (1) **Merge data** of all dose-response experiments.
 - (2) Lee and Kong procedure: replace simple linear regression model by **linear mixed effects model**.
 - Accounts for variability between experiments.
 - Yields reliable estimates of the interaction index.
 - Confidence bounds for curve of estimated interaction indices will be **wide** in case of **few** experiments with **large** between-experiment variability.

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References

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