p-value, s-value, b-value, d-value,... what else ?

> The probability of being out of specification ! An Ode to Tolerance



Bernard G Francq, Ron S Kenett NCS conference 2022

Introduction – Medical Research

Which of the following sentences would you prefer your surgeon to tell you?



PS: If you struggle to read this text until the end, then you better check your eyes with your ophthalmologist

- Surgery A is significantly better than B, on average (p<0.001)
- If you undergo surgery A, and your friend surgery B, then there is at most 20% chance that you end up with a better clinical outcome
- If you undergo surgery A instead of B, there is at most 30% chance that you end up with a better clinical outcome

Introduction – Significance Crisis Traditional null-hypothesis significance-testing...

- 1963: "no longer a sound of fruitful basis for statistical investigation" (Clark)
- 1978: "radically defective as to be scientifically almost pointless" (Meehl)
- 1978: "should be eliminated; it is harmful" (Carver)
- 1987: "despite two decades of attacks, the mystifying doctrine of null hypothesis is still today the Bible" (Gigerenzer and Murray)
- 1994: "hypothesis testing does not tell us what we want to know... out of desperation, we nevertheless believe that it does" (Cohen)
- 2003: "null hypothesis testing can actually impede scientific progress" (Kirk)

Mark Burgman (Imperial College London) What should applied science journal editors do about statistical controversies?

The debate is quite 'popular' nowadays

- 2016: The ASA statement on p-values: context, process, and purposes (Wasserstein and Lazar, The American Statistician)
- 2018: Statistical Inference as Severe Testing: How to get beyond the Statistics Wars (Mayo)
- 2019: Moving to a world beyond "p < 0.05" (Wasserstein et al., The American Statistician)
- 2019: valid p-values behave exactly as they should: some misleading criticisms of p-values and their resolution with <u>s-values</u> (Greenland, The American Statistician)
- 2019: Scientists rise up against statistical significance (Amrhein et al., Nature)
- 2020: "To claim a result to be highly significant, or even just significant, sounds like enthusiastic endorsement, whereas to describe a result as insignificant is surely dismissive" (Sir David Cox, Annu. Rev. Stat. Appl.)

Significance Crisis: our contribution

"Individual Success Probability (ISP): Beyond the t-test and p-values" (2022, under review)

<u>Our work</u>

- Smart Risk in CMC and Non-Clinical Statistics
- Compare the p-values, s-values, b-values, d-values, Probability Indexes, Generalized Pairwise Comparison
- Assess the uncertainty of the b-value and Probability Indexes
- Propose the ISP concept by 'reversing' the tolerance interval concept (not well known in clinical statistics)
- Show the one-to-one function p-value ISP

Overview of presentation

Smart Risk

Medical Research: misinterpretations

1 sample t-test

- Significance tests: Confidence Interval (CI), p-value, s-value
- Prediction Interval (PI)
- <u>Tolerance</u> Interval (TI)
- Individual Success Probability (ISP)
- p-value or ISP

Measurement error

2-arms clinical trials (B-value)

Smart Risk in CMC Statistics Probability Out Of Specification (POOS)

Question

How to quantify the probability of being out of specification (POOS)? What is the probability to be greater than the upper limit? ie a future lot, batch, product,...

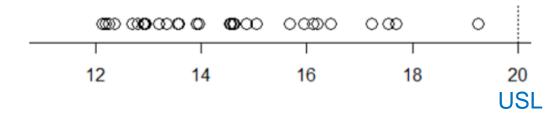
Parameters

- Upper Specification Limit (USL) = 20
- Type I Error = $\alpha = 0.05 = 5\%$
- Maximal *tolerated* OOS = 2%
- Sample Size = n = 30

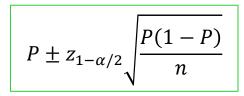
Assumptions

Normality, independence,...

Data Visualisation



What is the proportion of products greater than 20? Naive approach: P = 0/30 = 0%but the 95% classical CI assumptions fails: [0,0]



Smart Risk in CMC Statistics Probability Out Of Specification (POOS)

What could be the proportion of future products greater than 20?

Solution (when P = 0) Use the "rule of 3": 3/nThe 95% CI is then [0, 0.1] \rightarrow at most 10% of products OOS!

Other solutions (in R) (P = 0 or $P \neq 0$)

	method	х	n	mean	lower	upper
1	agresti-coull	0	30	0.000	-0.021	0.135
2	asymptotic	0	30	0.000	0.000	0.000
3	bayes	0	30	0.016	0.000	0.062
4	cloglog	0	30	0.000	0.000	0.116
5	exact	0	30	0.000	0.000	0.116
6	logit	0	30	0.000	0.000	0.116
7	probit	0	30	0.000	0.000	0.116
8	profile	0	30	0.000	0.000	0.104
9	lrt	0	30	0.000	0.000	0.062
10	prop.test	0	30	0.000	0.000	0.141
11	wilson	0	30	0.000	0.000	0.114

library(binom)

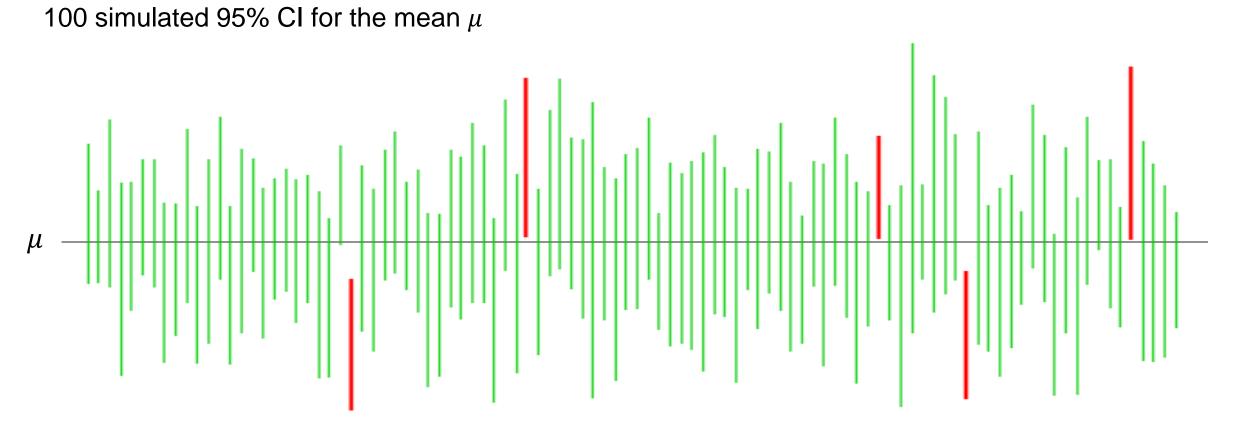
USL = 20 x = rnorm(30) x = (x - mean(x)) / sd(x) * 1.87 + 14.51binom.confint(x = sum(x > USL), n = length(x), conf.level = 0.95)

Interpretation

The upper bounds are all close to 10%The maximal ,threshold' was $2\% \rightarrow$ Stop!

Can we do better? Calculate Statistical Intervals...

Confidence Interval concept

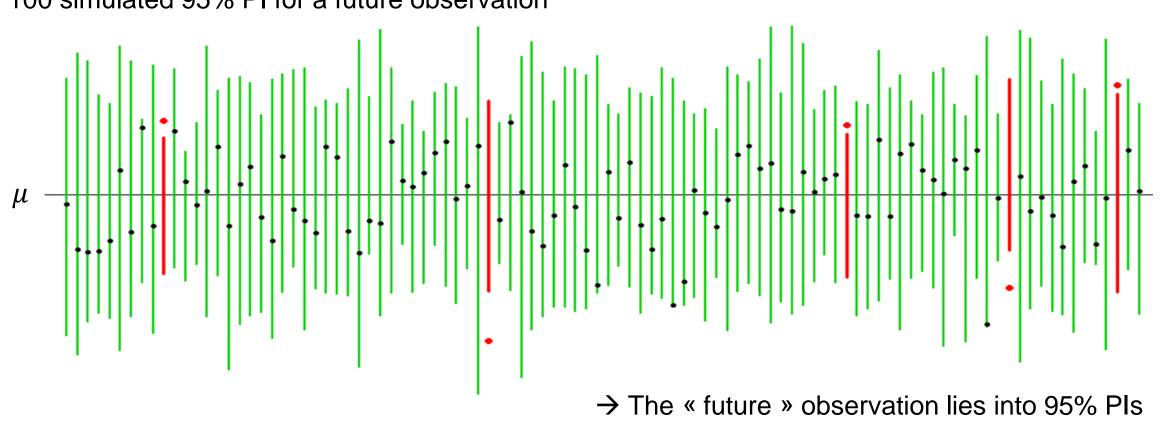


 \rightarrow The true value, μ , lies in 95% of the CIs

Note: in Bayesian statistics, credible intervals are usually used

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Prediction Interval concept

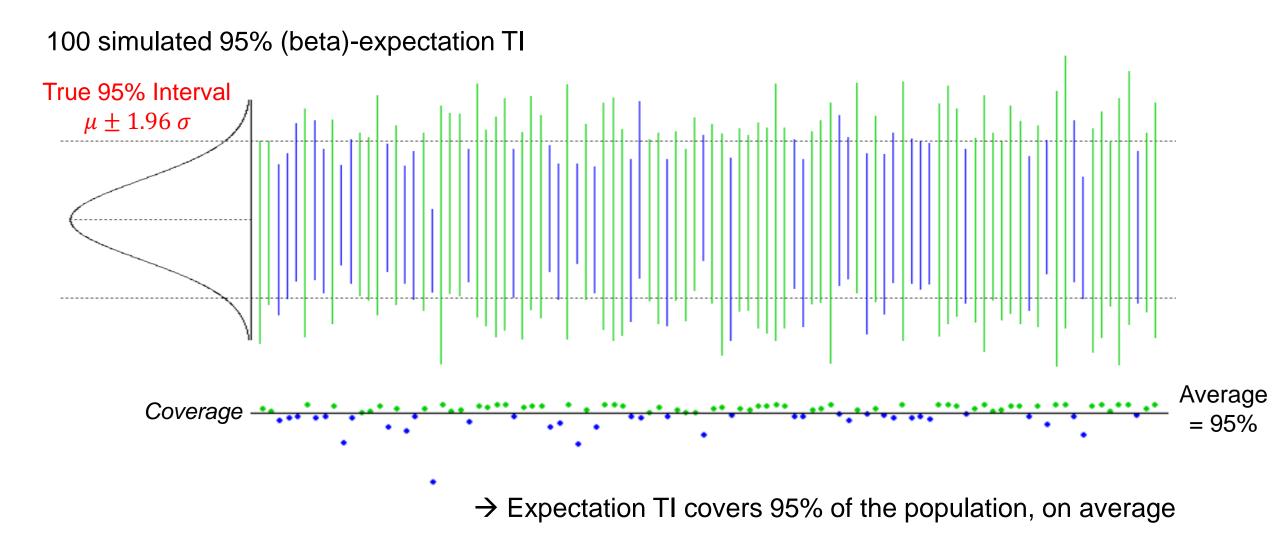


100 simulated 95% PI for a future observation

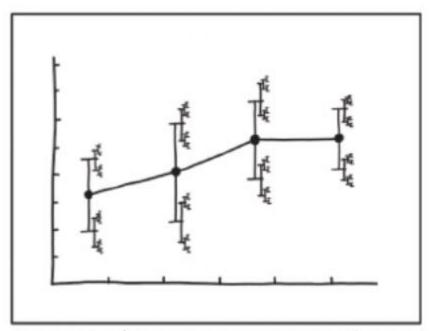
Note: in Bayesian statistics, PI can be obtained from the posterior distribution

NC

Expectation Tolerance Interval (type I) concept



Confidence Interval of Confidence Interval



I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.

https://www.explainxkcd.com/ Error Bars

• Will the PI contain less or more than 95% of future observations?

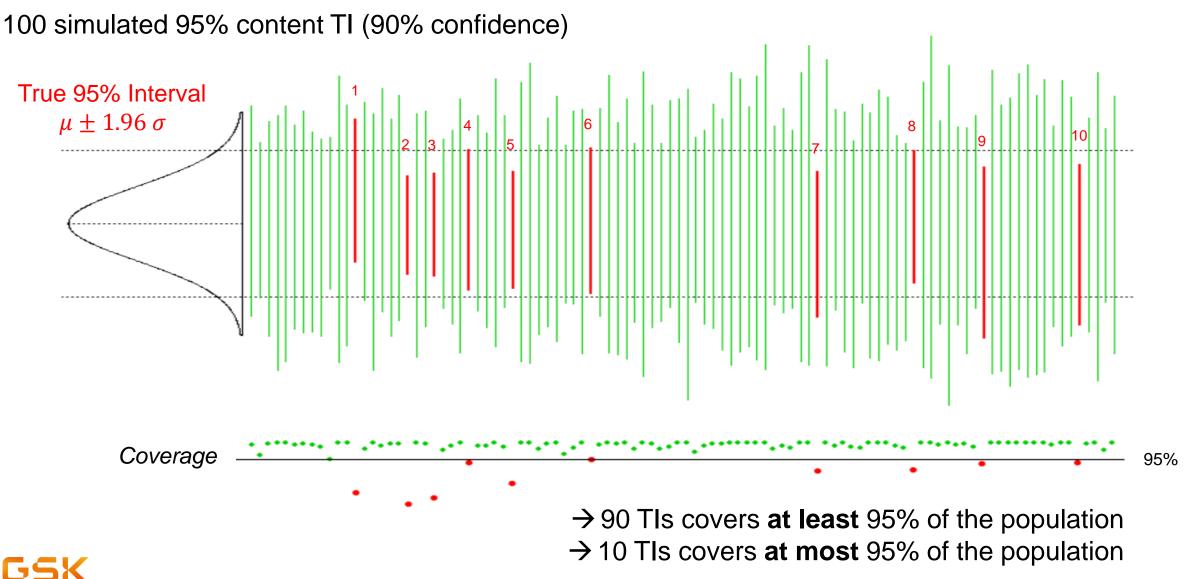
→ Some researchers calculate the 95% CI for each bound of the 95% PI

- Calculating the CI of a CI is awkward, confusing, misleading
- Unfortunately, widely used in method comparison studies (bridging studies) with Bland-Altman plot (agreement interval)

 \rightarrow Use the Tolerance Interval type II *

* To tolerate or to agree: <u>A tutorial on tolerance intervals</u> in method comparison studies with BivRegBLS R Package BG Francq, M Berger, C Boachie, *Statistics in Medicine*, 2020.

Content Tolerance Interval (type II) concept



Exact 1-sided Tolerance Intervals

TIs encompass a given proportion of the population with a given confidence level

The exact 1-sided TI is given by the <u>non-central t-distribution</u>

$$\overline{K} \pm t_{conf,n-1,\mathbf{Z}_{pred}\sqrt{n}} \frac{S}{\sqrt{n}}$$

- - or + must be chosen according to the context
- *conf* is the desired confidence level
- *pred* is the desired prediction level (coverage)
- n-1 are the degrees of freedom
- $z_{pred}\sqrt{n}$ is the non-centrality parameter
- *z*_{*pred*} is the quantile of the standardized normal distribution

TI and quantiles

A 1-sided TI is **identical** to calculating a 1-sided Confidence Interval on a quantile

Confidence, Prediction and Tolerance

90% CI		90% PI		98% TI (95% Conf)	
13.92	15.09	11.27	17.74	-∞	19.61

Confidence Interval = CI

• The interpretation is usually confusing and holds only for the average

Prediction Interval = PI

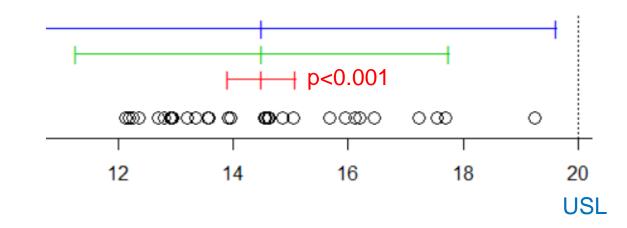
• A future product is expected to be between 11.27 and 17.74 (with 90% confidence)

(β)-expectation Tolerance Interval = TI type I

• 90% of the future products are expected to be between 11.27 and 17.74 (on average)

 $(\beta\gamma)$ -content Tolerance Interval = TI type II

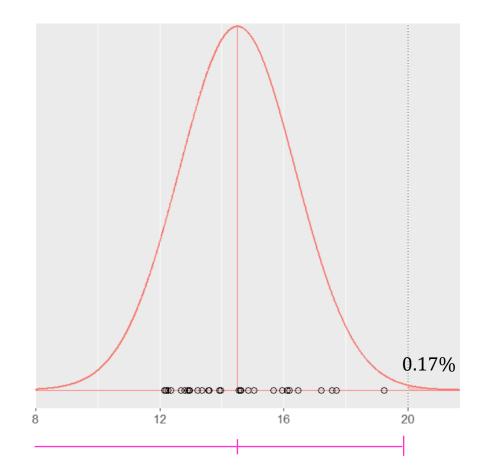
- At least 98% of the future products will be lower than 19.61 (with 95% confidence)
- Remarks The interpretation of PI and TI is similar in frequentist or Bayesian
 - Their interpretation remains identical with/without the log transformation



→ TI < USL
→ The POOS is lower than 2%
→ Smart Risk decision: Go ☺

Can we calculate the POOS?

Smart Risk: Probability Out Of Specification (POOS)



$$1 - \phi\left(z = \frac{USL - \bar{X}}{S}\right) = 0.17\%$$

Interpretation We expect 0.17% products OOS

95% Upper Bound by 'reversing' the TI $14.51 + t_{0.95,29, \mathbf{z}_{pred}} \sqrt{30} \frac{1.87}{\sqrt{30}} = 20$

uniroot(function(k) mean(x) + qt(0.95, 29, qnorm(k)*sqrt(n)) * sd(x) / sqrt(n) - 20,)

At least 98.7% of future products will be lower than 20 (95% confidence)

At most 1.3% will be greater than 20 (95% confidence) The POOS is 0.17% with a 95% upper bound equal to 1.3%

This « worst » scenario is lower than the maximal OOS (2%) \rightarrow Smart Risk Decision: Go \odot

Smart Risk: Summary

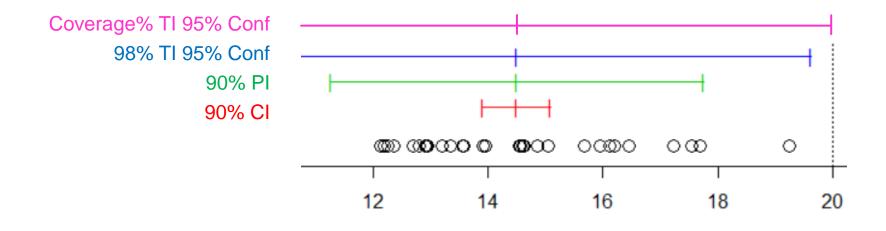
Calculate the TI with the desired confidence level, the prediction (coverage) is related to the maximal tolerated risk

If the TI does not overlap the 'threshold'

- \rightarrow Go \odot
- \rightarrow Otherwise, stop

Or, calculate the POOS and its upper bound If the upper bound exceeds the maximal tolerated POOS

- → Stop
- \rightarrow Otherwise, Go \odot





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Smart Risk

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2-arms clinical trials (B-value)

Medical Research: p-values and confidence intervals (CIs)

p-values and CIs are common in medical research and requested by most of top medical journals

- Curieusement la méta-analyse Fiolet montre une réduction significative de la mortalité de 17% :
- * HR = 0,83 [0,65-1,06], p<0,01.
- Pourtant les auteurs concluent à l'inefficacité de l'HCQ !!!

▶ 🜒 31:34 / 40:00

https://www.youtube.com/watch?v=FIhp7PtCEtY Critical analysis of treatments for COVID-19 (Analyse critique des traitements de la COVID-19)

HCQ is effective for COVID-19 when used early: real-time meta analysis of 205 studies

Corpus ID: 231610073, Published 2021

- HCQ is effective for COVID-19. The probability that an ineffective treatment generated results as positive as the 205 studies to date is estimated to be 1 in 28 quadrillion (p = 0.000000000000036).
- Studies from North America are 3.7 times more likely to report negative results than studies from the rest of the world combined, p = 0.0000022.

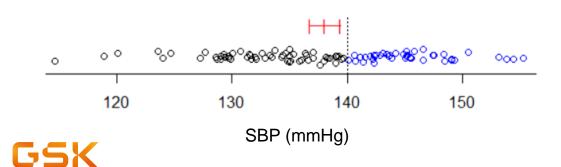
Researchers proud to show tiny p-values

1-sample t-test Toy Example: systolic blood pressure (SBP) (mmHg)

$$H_0: \mu = 140$$
 $\alpha = 0.05$
 $H_1: \mu < 140$ $n = 100$

The mean is estimated by $\overline{X} = 138.11$ and the standard deviation by S = 7.97

The 90% CI for the mean is therefore: [136.79, 139.43] The 1-sided 95% CI for the mean is: $]-\infty, 139.43]$



The estimated mean is < 140Its uncertainty as well: $140 \notin CI$

→ H_0 is rejected → The mean is significantly lower than 140 mmHg

1-sample t-test p-value, s-value

 $H_0: \mu = 140$ $H_1: \mu < 140$

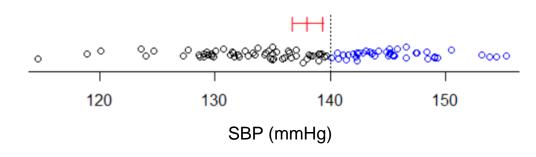
What about the p-value?

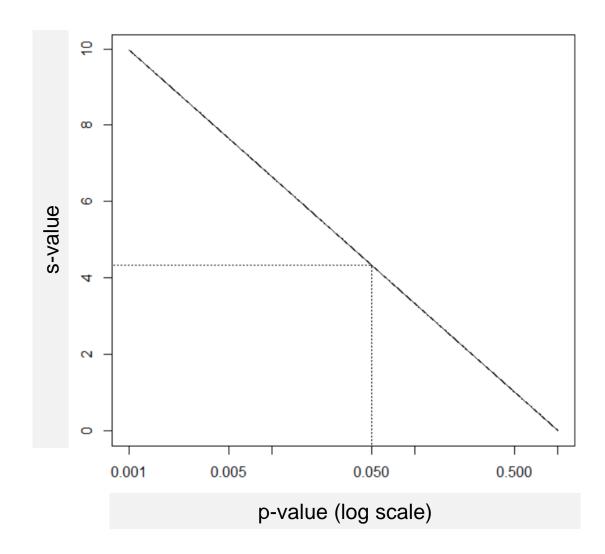
p-value = 0.0098 (significant at $\alpha = 0.05$)

What about the s-value?

```
s-value = -\log_2(p-value) = 6.7
```

The p-value is equivalent of obtaining more than 6 heads in a row when tossing a fair coin





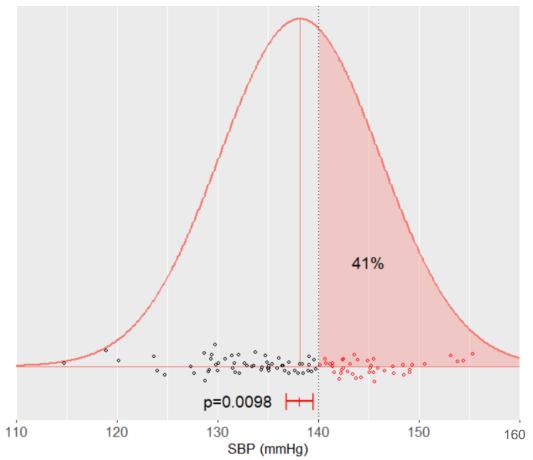
1-sample t-test Individual Success Probability

 $H_0: \mu = 140$ $H_1: \mu < 140$

What about the proportion of patients with a SBP > 140 ?

$$\phi\left(\frac{\bar{X}-140}{S}\right) = 41\%$$

This will be called, here, the Individual Success Probability (ISP)





1-sample t-test CI, p-value, s-value, ISP

What if the sample size increases (with identical mean and SD)?

				$H_1: \mu < 140$		ISP (Probat	oility Index)			
n	$\overline{\mathbf{X}}$	S	90% CI	p-value	s-value # Head	<i>P</i> (X < 140)	P(X > 140)			
20	138.11	7.97	[135.0, 141.2]	p=0.15	2.7	59.4%	40.6%			
50	138.11	7.97	[136.2, <mark>140.0</mark>]	p=0.05	4.3	59.4%	40.6%			
100	138.11	7.97	[136.8, 139.4]	p=0.0098	6.7	59.4%	40.6%			
200	138.11	7.97	[137.2, 139.0]	p=5E-4	11	59.4%	40.6%			
10 ³	138.11	7.97	[137.7, 138.5]	p=7E-14	44	59.4%	40.6%			
	p<0.001 How to quantify the The p-values collapse (the s-values soar) while the ISP remains constant uncertainty on the ISP?									

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1-sample t-test

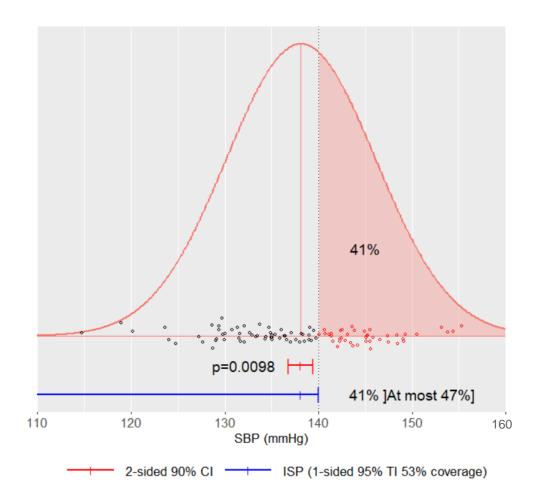
How to take into account the uncertainty on the ISP?

What should be the value of the prediction level (coverage) for the TI to be equal to 140 ?

$$138.11 + t_{0.95,100-1,z_{pred}\sqrt{100}} \frac{7.97}{\sqrt{100}} = 140$$

• At most 47% of the patients have a SBP > 140

 \rightarrow This is the 95% upper (lower) bound for the ISP



1-sample t-test Toy Example: systolic blood pressure (SBP) (mmHg)

What if the sample size increases (with identical mean and SD)?

				$H_1: \mu < 140$		ISP (9	5% CI)
n	$\overline{\mathbf{X}}$	S	90% CI	p-value	s-value # Head	P(X < 140)	P(X > 140)
20	138.11	7.97	[135.0, 141.2]	p=0.15	2.7	59.4 [44.5[%	40.6]55.5]%
50	138.11	7.97	[136.2, 140.0]	p=0.05	4.3	59.4 [50.0[%	40.6]50.0]%
100	138.11	7.97	[136.8, 139.4]	p=0.0098	6.7	59.4 [52.8[%	40.6]47.2]%
200	138.11	7.97	[137.2, 139.0]	p<0.001	11	59.4 [54.7[%	40.6]45.3]%
10 ³	138.11	7.97	[137.7, 138.5]	p<0.001	44	59.4 [57.3 [%	40.6] 42.7]%

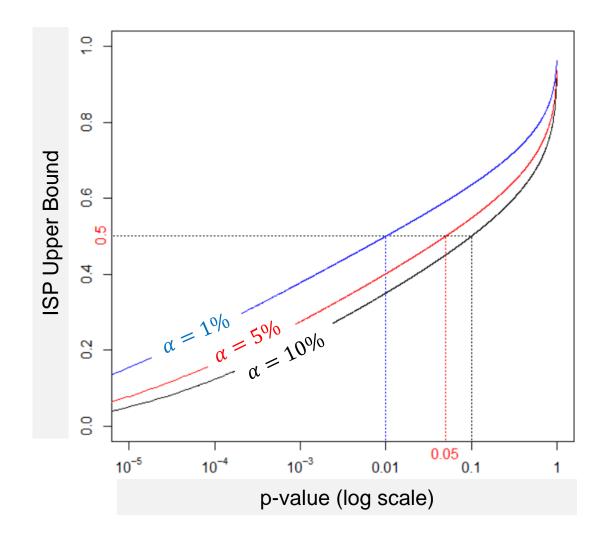
CI and p-value might be confusing

The ISP interpretation is straightforward even for big sample sizes (eg $n = 10^3$)

✓ At least 57.3% of the (new) patients will have a SBP <140 mmHg (success)

✓ At most 42.7% of the (new) patients will have a SBP >140 mmHg (failure)

One-to-one function ISP & p-value



 $X \sim N(\mu = 145, \sigma = 5)$ n = 10 $H_0: \mu = 140, H_1: \mu > 140$

The (upper bound) ISP is a one-to-one function with the p-value

Main advantages of the ISP over the p-value

- ✓ Easy to interpret
- \checkmark No tiny values
- ✓ No need to use sophisticated rounding rules
- ✓ Realistic and pragmatic interpretation
- ✓ Similar interpretation *frequentist* and *Bayesian*
- ✓ Identical interpretation for log or no-log data
- ✓ The cut-off value is 50% (the middle of the probability scale), an intuitive threshold, whatever the type I error

Overview of presentation

Smart Risk

Medical Research: misinterpretations

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Measurement error

2-arms clinical trials (B-value)

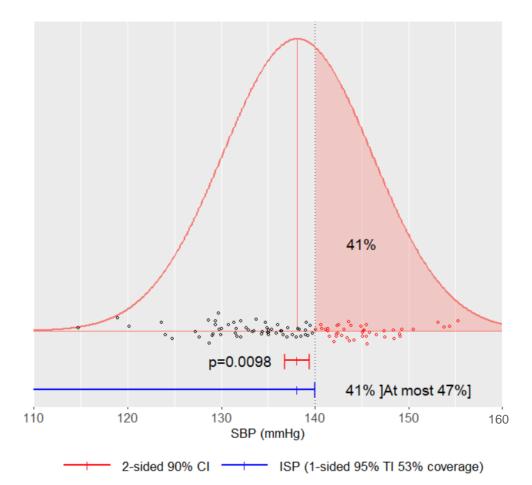
ISP and measurement error

The SBP is certainly measured with some measurement error

→ What is the probability for the 'true' SBP to be > 140 ? ('true' = without measurement error)

Define more precisely, clarify the desired ISP:

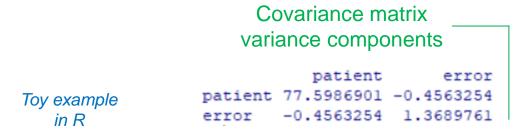
- $P(X_T > 140)$ where X_T is the 'true' value of the next patient
- P(X > 140) where X is the SBP measured on the next patient



- ISP and measurement error: use replicates

- n = 50 patients, each measured 3 times
- Mixed model by REML method
- Between variance and within variances are the 2 key parameters

```
Individual fixef effect estimates:
Estimate Std. Error Lower Upper
(Intercept) 137.0483 0.9313913 135.1766 138.92
Variance component estimates:
patient error
40.619634 8.264555
```



ISP and measurement error

The ISP is assessed by the z-score and by using the corresponding variance components. Example for P(X > 140) with the total variance

$$P(X > 140) = 1 - \phi\left(z = \frac{140 - \hat{\mu}}{\hat{\sigma}_T}\right)$$

The lower and/or upper bounds can be obtained by the delta method on the z-score *

$$CI \{ P(X > 140) \} = 1 - \phi \left(\frac{140 - \hat{\mu}}{\hat{\sigma}_T} \pm z_{0.95} \sqrt{var(z)} \right)$$

If needed (especially for small sample sizes), $z_{0.95}$ can be replaced by the t-distribution with the DF as:

- (Kenward-Roger)
- (Satterthwaite)
- ✓ Francq et al. **

 $P(X_T > 140)$ is assessed with the between variance



* Delta Method and Bootstrap in Linear Mixed Models to Estimate a Proportion When no Event is Observed: Application to Intralesional Resection in Bone Tumor Surgery. BG Francq, O Cartiaux. *Statistics in Medicine* (2016)

29

ISP, measurement error and Smart Risk

Individual fixef effect estimates:		
Estimate Std. Error Lower Upper		
(Intercept) 137.0483 0.9313913 135.1766 138.92		Covariance matrix
		variance components
Variance component estimates:		
patient error		patient error
patient critic	Toy example	patient 77.5986901 -0.4563254
40.619634 8.264555	in R	error -0.4563254 1.3689761

- *P*(*X* > 140) = 33.6]44.0]% At most 44% of future patients will have their SBP measured > 140
- $P(X_T > 140) = 32.2$]43.6]% At most 43.6% of future patients will have their 'true' SBP > 140

Smart Risk ——	
Small RISK	What matters is
	What matters is
the probability	that a future product has its true (underlying) value outside the spec
	(and not its measured value)



Overview of presentation

Smart Risk

Medical Research: misinterpretations

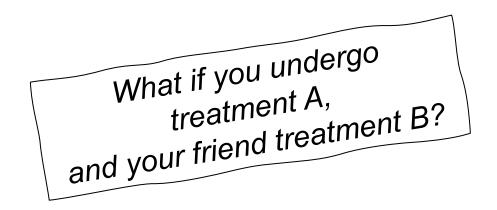
1 sample t-test

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Measurement error

2-arms clinical trials (B-value)

- 2-arms clinical trials: Parallel treatment groups



- Aⁿ⁺¹ is the outcome of a new patient under treatment A
- Bⁿ⁺¹ is the outcome of a new patient under treatment B

Aⁿ⁺¹ and Bⁿ⁺¹ are independent

→ $P(A^{n+1} > B^{n+1})$ is the ISP (for A over B) → $P(B^{n+1} > A^{n+1})$ is the ISP (for B over A)

<u>Remark</u> $P(A^{n+1} > B^{n+1})$ is here simplified as P(A > B), and also named the « B-value » in the literature (or D-value when reversed) *

- 2-arms clinical trials: Summary

			Mean	$H_0:\mu_D=0$		$(\mathbf{p} < 0)$	(D > 0)
	Mean	Pooled	Difference	$H_1: \mu_D$	≠ 0	P(D < 0)	P(D > 0)
n	Diff.	SD	95% CI	p-value	# Head	b-value	d-value
50	0.12	1.41	[-0.27, 0.52]	p=0.54	0.9	53.5	46.5
100	0.12	1.41	[-0.15, 0.40]	p=0.38	1.4	53.5	46.5
500	0.12	1.41	[0, 0.25]	p=0.05	4.3	53.5	46.5
1000	0.12	1.41	[0.04, 0.21]	p=0.006	7.5	53.5	46.5
5000	0.12	1.41	[0.08, 0.16]	p<.001	31	53.5	46.5
				p-value collapse	s-value soar	b-value constant	d-value constant

How to add the 95% CI?

✓ Reverse the Tolerance Interval for a Difference !

 ✓ Well-established methodology in non-clinical statistics Example: comparability,...

- 2-arms clinical trials: Summary

			Mean	$H_0:\mu_D=0$				Success I	Probability
	Mean	Pooled	Difference	$H_1: \mu_D$	≠ 0			P(D < 0)	P(D > 0)
n	Diff.	SD	95% CI	p-value	# Head	b-value	d-value	95% CI	95% CI
50	0.12	1.41	[-0.27, 0.52]	p=0.54	0.9	53.5	46.5	[42.5, 64.2]%	[35.8, 57.5]%
100	0.12	1.41	[-0.15, 0.40]	p=0.38	1.4	53.5	46.5	[45.7, 61.2]%	[38.8, 54.3]%
500	0.12	1.41	[0, 0.25]	p=0.05	4.3	53.5	46.5	[50.0, 57.0]%	[43.0, 50.0]%
1000	0.12	1.41	[0.04, 0.21]	p=0.006	7.5	53.5	46.5	[51.0, 56.0]%	[44.0, 49.0]%
5000	0.12	1.41	[0.08, 0.16]	p<.001	31	53.5	46.5	[52.4 , 54.6]%	[45.4, 47.6]%
				p-value collapse	s-value soar	b-value constant	d-value constant	L	

ISP interpretation (n = 5000) *

- × At least 52.4% patients are expected to be better with B (than A)
- At least 52.4% patients are expected to get a better clinical outcome with treatment B compared to their friends under A
- By comparing A and B on different patients, B is expected to be better in at least 52.4% of the comparisons

Borderline test (n = 500)

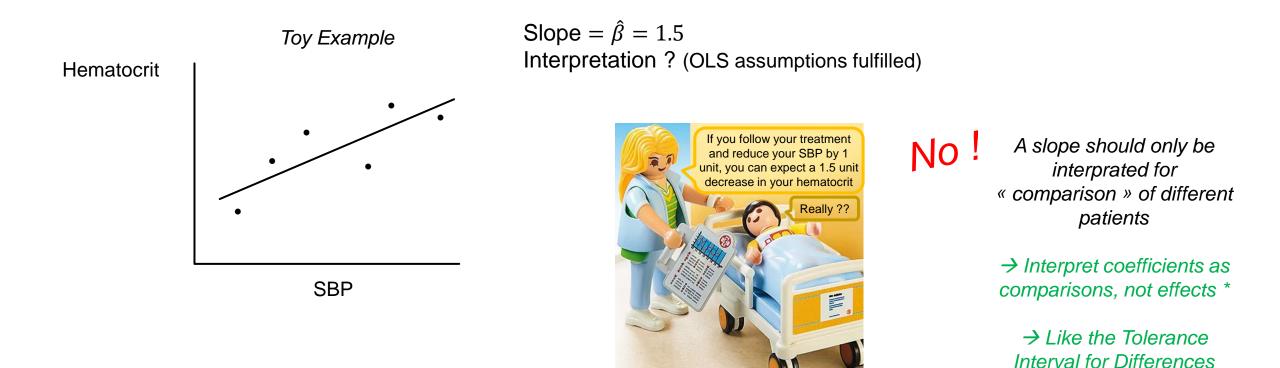
• p-value = 5%

- CI bound = 0
- ISP bound = 50%

GSK

* S Greenland *et al.* On Causal Inferences for **Personalized Medicine**: How **Hidden Causal Assumptions Led to Erroneous Causal Claims About the D-Value**. *The American Statistician* 2019. Demystify a (statistical) urban legend

How do you interpret a slope ?



GSK

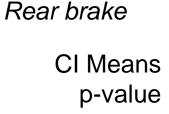
* A Gelman, J Hill, A Vehtari. Regression and Other Stories. Published by Cambridge University Press in 2020.

When you bike, do you mainly use the front break or the rear one?

Front brake

Prediction Tolerance ISP Bayesian,...





Majority of people mainly use the rear brake, because we learnt it. We actually have to use the front brake !

While CI and p-value can be confusing or controversial, Smart Risk, Tolerance Intervals and Success Probabilities have straightforward interpretations

GSK



References

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Acknowledgment

Dan Lin Walter Hoyer Projects CMC Stat Team at GSK

Conflict of interest

This work was sponsored by GlaxoSmithKline Biologicals SA. BG Francq is employee of the GSK group of companies. R Kenett is an employee of the KPA group and the Samuel Neaman Institute.

- 2-arms clinical trials: Summary

			Mean	$H_0:\mu_D=0$				Success I	Probability
	Mean	Pooled	Difference	$H_1: \mu_D$	≠ 0			P(D < 0)	P(D > 0)
n	Diff.	SD	95% CI	p-value	# Head	b-value	d-value	95% CI	95% CI
50	0.12	1.41	[-0.27, 0.52]	p=0.54	0.9	53.5	46.5	[42.5, 64.2]%	[35.8, 57.5]%
100	0.12	1.41	[-0.15, 0.40]	p=0.38	1.4	53.5	46.5	[45.7, 61.2]%	[38.8, 54.3]%
500	0.12	1.41	[0, 0.25]	p=0.05	4.3	53.5	46.5	[50.0, 57.0]%	[43.0, 50.0]%
1000	0.12	1.41	[0.04, 0.21]	p=0.006	7.5	53.5	46.5	[51.0, 56.0]%	[44.0, 49.0]%
5000	0.12	1.41	[0.08, 0.16]	p<.001	31	53.5	46.5	[52.4 , 54.6]%	[45.4, 47.6]%

Our reply

The d-value you can't buy...

"Individual Success Probability: Beyond the t-test and p-values" (2022, under review)

Eugene Demidenko. **The p-value you can't buy.** *The American Statistician* 2016; 70: 33 – 38.