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A Comparative Analysis of Bayesian Sampling Methodologies for Design Space Identification in Quality by Design (QbD) Approach

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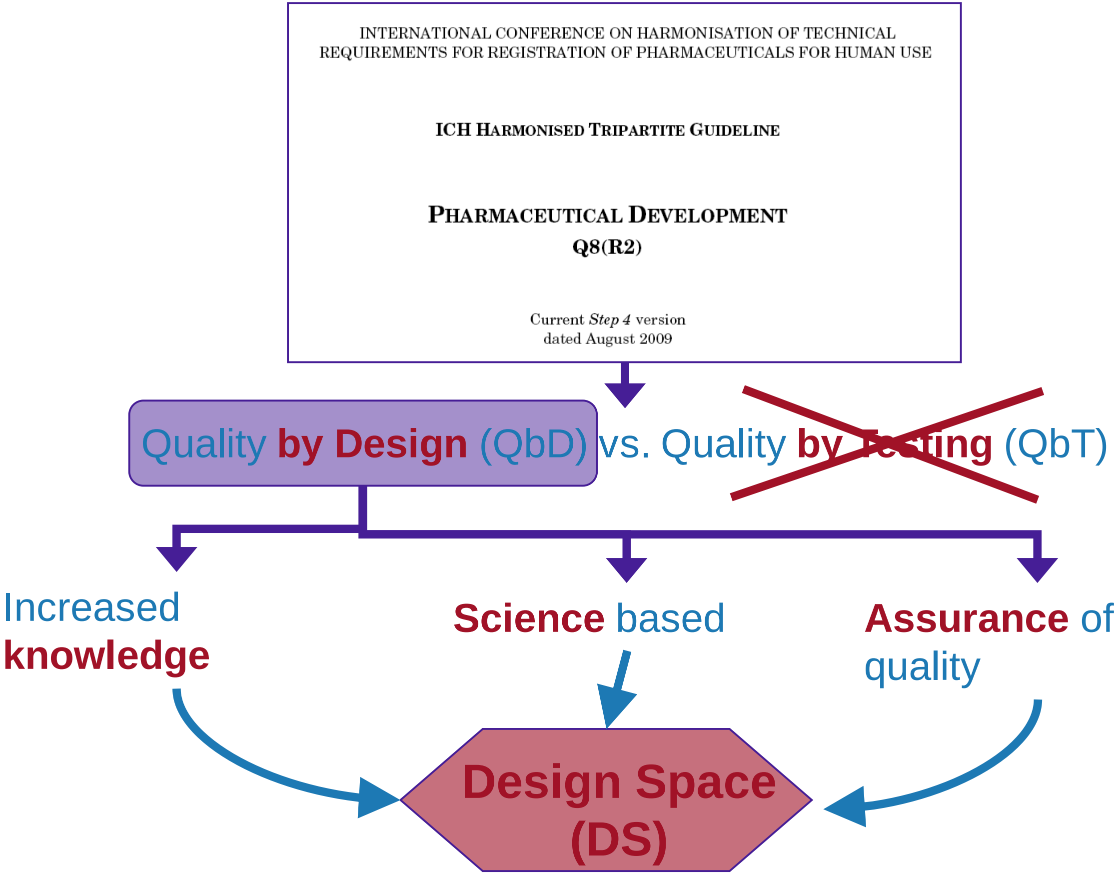
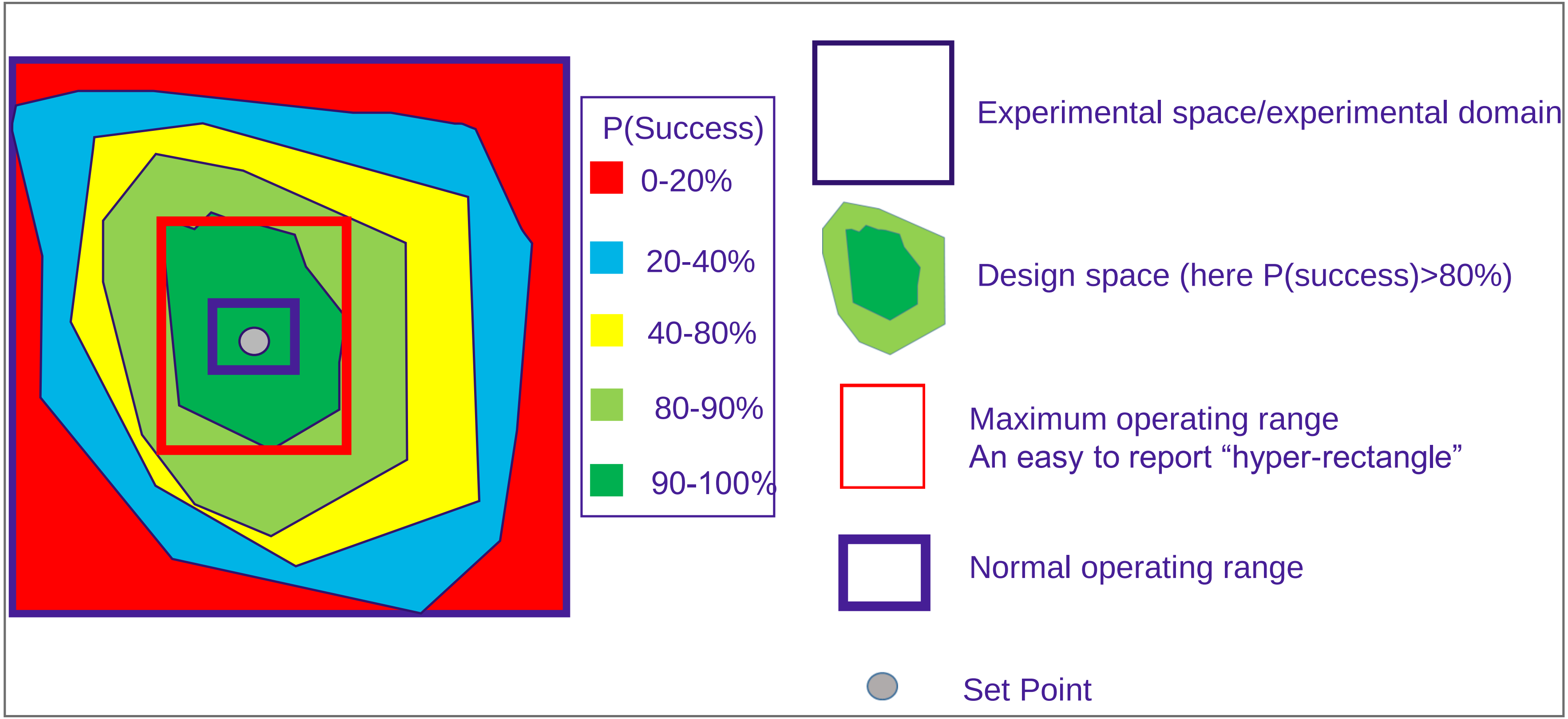
Design Space Identification in QbD approach



Design Space Identification in QbD approach

Data → Fit Bayesian Model → Generate a large grid using a space filling design algorithm (~1000 -5000 points) → Using posterior distributions and for each row of the grid, compute the probability of success to meet specifications → Plot results using a pair plot → Visual Identification of the Design Space

Design Space - 2 factors example



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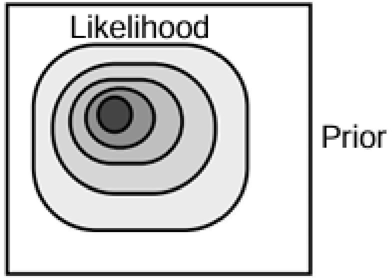
Background

$$\underbrace{\Pr(\boldsymbol{\Theta}|\mathbf{D},\mathrm{M})}^{\text{Posterior}} = \frac{\overbrace{\Pr(\mathbf{D}|\boldsymbol{\Theta},\mathrm{M})}^{\text{Likelihood}}\ \overbrace{\Pr(\boldsymbol{\Theta}|\mathrm{M})}^{\text{Prior}}}{\underbrace{\Pr(\mathbf{D}|\mathrm{M})}_{\text{Evidence}}}$$

Bayes' Theorem

Motivation: Sampling Posterior

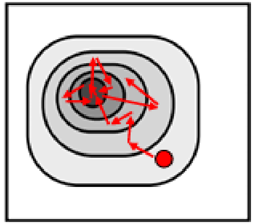


Sampling directly from the likelihood $\mathcal{L}(\Theta)$ is **hard**.

MCMC: Solving a Hard Problem once.
(Markov Chain Monte Carlo)

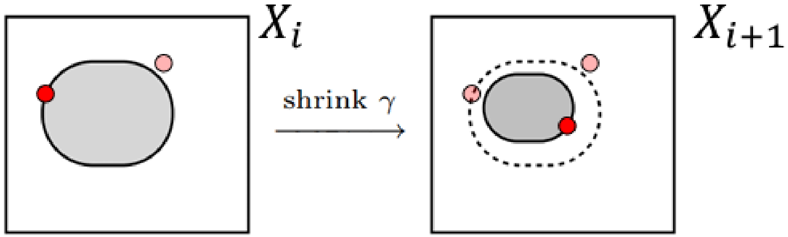
Source: Speagle, Joshua S. (2021)

Motivation: Sampling Posterior



MCMC: Solving a Hard Problem once.
vs
Nested Sampling: Solving an Easier Problem many times.

Sampling $\boxed{\text{uniformly within}}$ bound $\mathcal{L}(\boldsymbol{\Theta}) > \lambda$ is **easier**.



Source: Speagle, Joshua S. (2021)

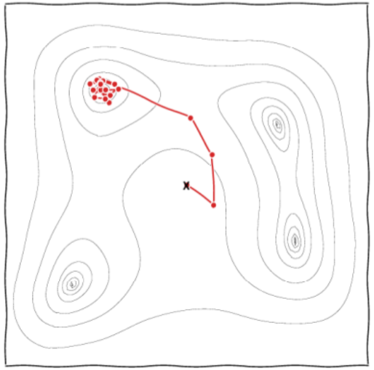
Comparison of Nested Sampling vs. MCMC Sampling

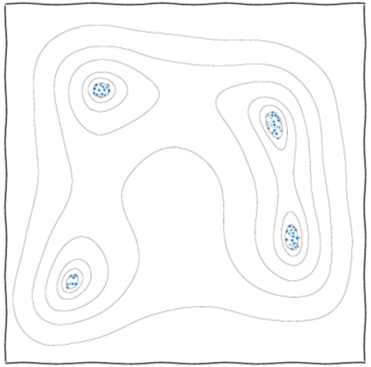
MCMC

- Single “walker”
- Explores posterior
- Fast, if proposal matrix is tuned

Nested Sampling

- Ensemble of “live points”
- Scans from prior to peak of likelihood
- Might be slower, no tuning required





Source: Handley, Will (2023)

Advantages and disadvantages of Nested Sampling

Advantages

- Does not need gradients
- Ensemble sampler
- Multimodal exploration
- Can characterize complex uncertainties in real-time
- Can allocate samples much more efficiently in some cases
- Very parallelizable
- Possesses well-motivated stopping criteria (Skilling 2006; Speagle 2020)
- Can help perform model selection

Disadvantages

- Can not use gradients as “naturally” as HMC.
- Implementations require a prior transform.
- Slow (but steady) and runtime sensitive to size of prior.
- Sampling is more involved.

The main goal of using gradients is to improve dimensionality scaling / reliability

Examples on Statistical Inference

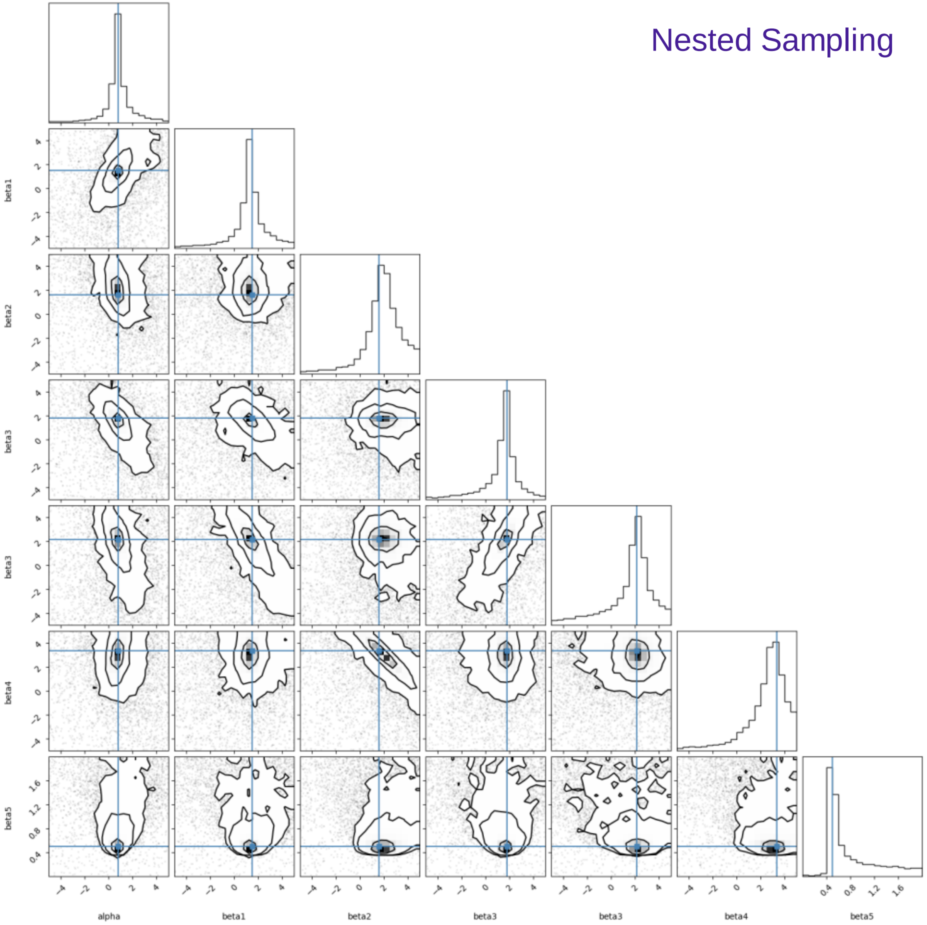
alpha	beta_1	beta_2	beta_3	beta_4	beta_5
0.811829	1.513266	1.590718	1.791521	2.132137	3.339008

$$Y_i = \alpha - \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 - \beta_4 X_1 X_3 + \beta_5 X_2 X_2 + \varepsilon_i$$

Computation Duration with dynesty <2.5min

Computation Duration with PyMC3 <9min

Nested Sampling



Example on Design Space Computation

For a simple case study with two design variables, d := (d1; d2) and a single CQA, s.

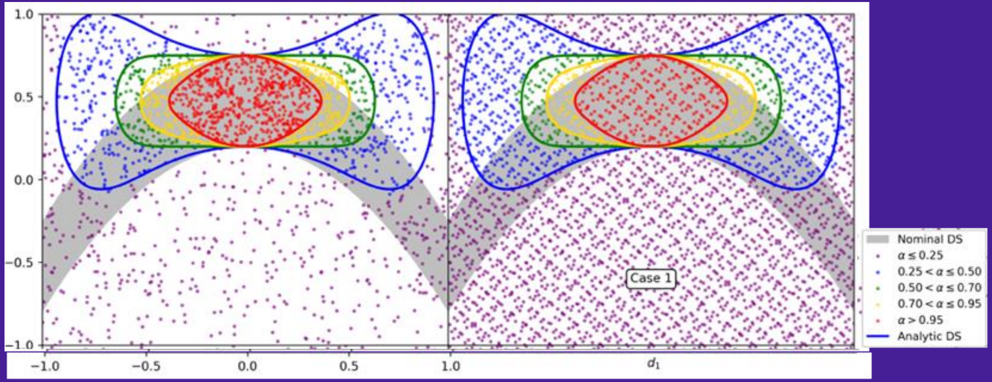
$$s = \theta d_1^2 + d_2$$

with the model parameter θ. The goal is to characterize the probabilistic DS inside the knowledge space K := [-1; 1]^2 imposed by the following CQA limits:

$$0.2 \leq s \leq 0.75$$

$$\mathcal{D}_{\text{nom}} := \{\boldsymbol{d} \in [-1,1]^2 : 0.20 \leq d_1^2 + d_2 \leq 0.75\}$$

$$\theta \sim \mathcal{N}(0,1)$$



$$\mathcal{D}_\alpha := \left\{\boldsymbol{d} \in [-1,1] : \mathbb{P}\left[0.20 \leq \theta d_1^2 + d_2 \leq 0.75 \,\middle|\, \theta \sim \mathcal{N}(\mu_\theta, \sigma_\theta)\right] \geq \alpha\right\}$$
$$= \left\{\boldsymbol{d} \in [-1,1] : \operatorname{erf}\left(\frac{0.75 - \mu_\theta d_1^2 - d_2}{\sqrt{2}\sigma_\theta d_1^2}\right) - \operatorname{erf}\left(\frac{0.20 - \mu_\theta d_1^2 - d_2}{\sqrt{2}\sigma_\theta d_1^2}\right) \geq 2\alpha\right\}$$

Reliability value	Samples drawn by standard Monte Carlo (Algorithm 1)	Samples drawn by nested sampling (Algorithm 2)
Uncertainty scenario: $\theta \sim \mathcal{N}(0,1)$		
$0.95 \leq \alpha$	247	500
$0.70 \leq \alpha < 0.95$	139	231
$0.50 \leq \alpha < 0.70$	172	181
$0.25 \leq \alpha < 0.50$	666	418
$\alpha < 0.25$	2,025	532
Total	3,249	1,862

Source: Kusumo, et al (2019)

Conclusions

To summarize, Nested Sampling is an attractive algorithm framework for Bayesian inference and design space computation because

- it explores the parameter space globally,
- it handles multi-modal distributions and phase transitions well,
- it initializes and terminates at a well-defined point without cumbersome supervision, and
- it leverages efficient strategies from Bayesian parameter estimation for generating replacement proposals during the DS characterization.
- it is effective for larger DS characterization with a handful of process parameters, in the presence of a complex dynamic model and realistic model uncertainty.
- it can even be competitive of MCMC with optimization methods relying on process flexibility concepts, even in low-dimensional DS characterization problems

Further Studies

- High Dimensional experiments:

- Advanced kinetic models based on ordinary differential equations

- Multimodal and mixture models

Thank you!

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3. Kusumo, K. P., Gomoescu, L., Paulen, R., García Muñoz, S., Pantelides, C. C., Shah, N., & Chachuat, B. (2019). Bayesian approach to probabilistic design space characterization: A nested sampling strategy. *Industrial & Engineering Chemistry Research*, *59*(6), 2396-2408.

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